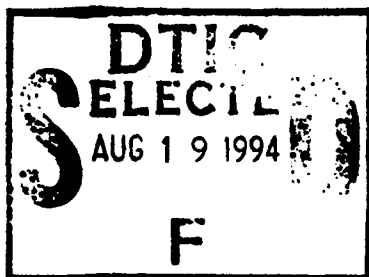


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THESIS

AN ANALYSIS OF DOD INSPECTOR
GENERAL'S STATISTICAL SAMPLING
PLAN FOR NAVY REPAIRABLE
ITEM PROCUREMENTS

by

Thomas D. Chase

June, 1994

Thesis Co-Advisors:

Alan W. McMasters
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by

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Lieutenant, United States Navy
B.S., Santa Clara University, 1983**

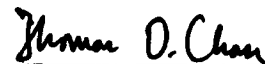
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ABSTRACT

A recent audit by the Department of Defense Inspector General (DODIG) of Navy inventory control points found a high value of purchase requests for repairable items that the auditors labeled as unnecessary or excessive. The dollar figures reported were based on the auditors' use of stratified sampling. This thesis examined the auditors' use of stratified sampling by attempting to replicate the auditors' process of stratifying and sampling. The author then attempted to verify the auditors' claimed confidence level and precision of the final result. This study questions the chosen sample size and sample stratification. In addition, this thesis found that the auditors' actual precision was not as tight as stated in the DODIG audit report. This was caused by the auditors' emphasis on the very high dollar value strata which had only a few purchase requests rather than on the stratum with the largest number of purchase requests. It was this latter stratum which had the highest projected number and total dollar value of excesses.

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I. INTRODUCTION

A. MOTIVATION

This study is based on the results of an economy and efficiency audit conducted by the Department of Defense Inspector General (DODIG) from August 1990 through November 1991 in accordance with standards issued by the Comptroller General of the United States (Jones, February 1, 1993, p. 2). The DODIG conducts audits of the Navy's inventory control points' (ICP) procurement actions using a multi-stage sampling plan that incorporates stratified sampling. Using these stratified samples, DODIG conducts audit tests to determine which purchase actions are for "reasonable" quantities.

When the results of this audit were reported, they also identified a total dollar value of items that were considered unnecessary or premature purchases. In this case, the DODIG identified potential monetary benefits totaling \$71.7 million, which represents the estimated value of unnecessary purchases. When audit findings disclose such "problem" purchases, obligational authority is reduced for the ICPs. The reasoning is that, if the funding had not been wasted on these particular procurement actions, the funding would not have been needed at all. In order to avoid inaccurate conclusions, it is absolutely critical that such audit findings accurately reflect weaknesses in inventory management.

B. OBJECTIVE

The focus of the thesis will be to look at how the DODIG incorporates stratified sampling methodologies in auditing major procurement actions and then making recommendations to inventory control points. In particular, this thesis will look at how the DODIG gathers the audit results from each stratum sampled and projects these findings over the entire population of procurement actions. This study explores the question of whether sampling stratification with current sample sizes can be used to make projections of potential monetary savings with a high confidence level and tight precision.

C. RESEARCH QUESTIONS

The primary question to be answered in this project is: "Does the DODIG's statistical sampling plan and stratified sampling of procurement actions lead to the most accurate conclusions and recommendations?" In order to answer this question completely, secondary questions concerning choices of stratifications, size of strata, and confidence levels within each stratum must be addressed. In particular, how is this sampling structure used to project errors in each stratum to the population as a whole?

D. SCOPE, LIMITATIONS, AND ASSUMPTIONS

The scope of this study is very narrowly defined in order to remain focused on the research questions mentioned above. In the audit report analyzed, there are many areas of debate that are clearly not resolved. Each one could be the basis for a separate study at a later date. This project will look at the basis for the amount of potential monetary benefits suggested in the audit report. As the report's Executive Summary states, the Navy disagrees with the basis for estimating potential monetary benefits (Jones, February 1, 1993, p. ii).

This project's primary focus is on how the DODIG used sampling to make its observations of the ICPs' procurement of repairable items. For example, this thesis considers the initial total quantity of line items, the smaller universe of line items chosen to sample, the adjusted universe of line items to sample, the sample of procurement actions to be scrutinized and the conclusions drawn from these samples. The area analyzed concerns the dollar value projection based on these findings. In statistical terminology, this thesis primarily focuses on the dollar value estimates of the unnecessary purchases reported in the audit to see if improvements could have been made and, secondarily, examines the frequency estimation. Dollar value and frequency estimation are defined in Chapter II, Section B.

This study is not intended to settle disagreements on terminology, determine whether the audit was poorly timed, nor make recommendations on how internal controls should be changed. Instead, this thesis concentrates on the statistical processes and on how they resulted, either accurately or inaccurately, in budgetary recommendations for the ICP's under review.

E. PREVIEW

The remaining chapters are organized in the following manner. Chapter II gives background information on stratified sampling, estimation sampling, confidence intervals, and the inventory management definition of stratification.

Chapter III presents all the data provided by the statisticians at the DODIG. The data is broken down as described in the audit report, and it shows the conclusions derived from this data. The chapter concludes with an explanation of how the audit results were actually used by the affected command when submitting a budget request

Chapter IV opens with a discussion of how the DODIG made its statistical decisions. The following section begins the author's analyses of the statistical decisions. The first analysis is of the selected universe size and sample size, followed by an alternative method to choose a sample size, and, finally, how to stratify the chosen sample size. The second major analysis is of the auditors' stated confidence level and related precision. The chapter closes with a comment on the several numerical discrepancies found in the audit report and working papers.

Chapter V summarizes the main points and conclusions of the thesis and presents several recommendations.

II. METHODOLOGY

A. STRATIFIED SAMPLING

1. Stratified Sampling Defined

In stratified random sampling, the population is divided into a series of independent subpopulations which are called "strata." Each stratum is sampled as though it were a separate population from which an unrestricted random sample were being drawn. It is then possible to combine the separate results to make a conclusion about the entire population. A degree of precision can then be applied to these results (Cyert, 1962, p. 116).

2. Why stratified sampling?

Auditors may not consider all accounts or records of equal importance. For instance, they may have a much greater interest in establishing the accuracy of large accounts and be unwilling to run as great a sampling risk for this type of account. Objective sampling methods do not necessarily require sampling from a general pool of items. It is not only possible but often desirable to segregate the population into separate groups, by size or other characteristics, and to sample with various degrees of accuracy in each area separately.

When sampling accounts, it is common to want to examine all of the large accounts, a large portion of the moderately sized accounts, and a relatively small proportion of the lowest dollar-valued accounts. In order to draw a clear conclusion from this process, it is necessary to stratify the accounts by size and then analyze the results. This procedure is looked upon favorably because there is improved sample reliability for the large accounts. Because it removes the large accounts from other sections of the sample, determination of the sampling variability also is more precise (Hill, 1962, p. 45).

3. How to Use Stratified Sampling

With stratified sampling of audit units, the heterogeneous population (Figure 1.A) is first divided into a number of mutually exclusive groups or strata (1.B). After the population of audit units has been divided into a number of mutually exclusive strata, a simple random sample (without replacement) of purchases is selected from each stratum (1.C). These samples must be independent, which can be accomplished by use of different sets of random numbers for the selection of the different random samples. After the samples are pulled from the strata, they are analyzed for specific attributes or values (1.D). Finally, based on the results of the analysis, a conclusion is drawn about the original population (1.E) (Neter, 1975, p. 77).

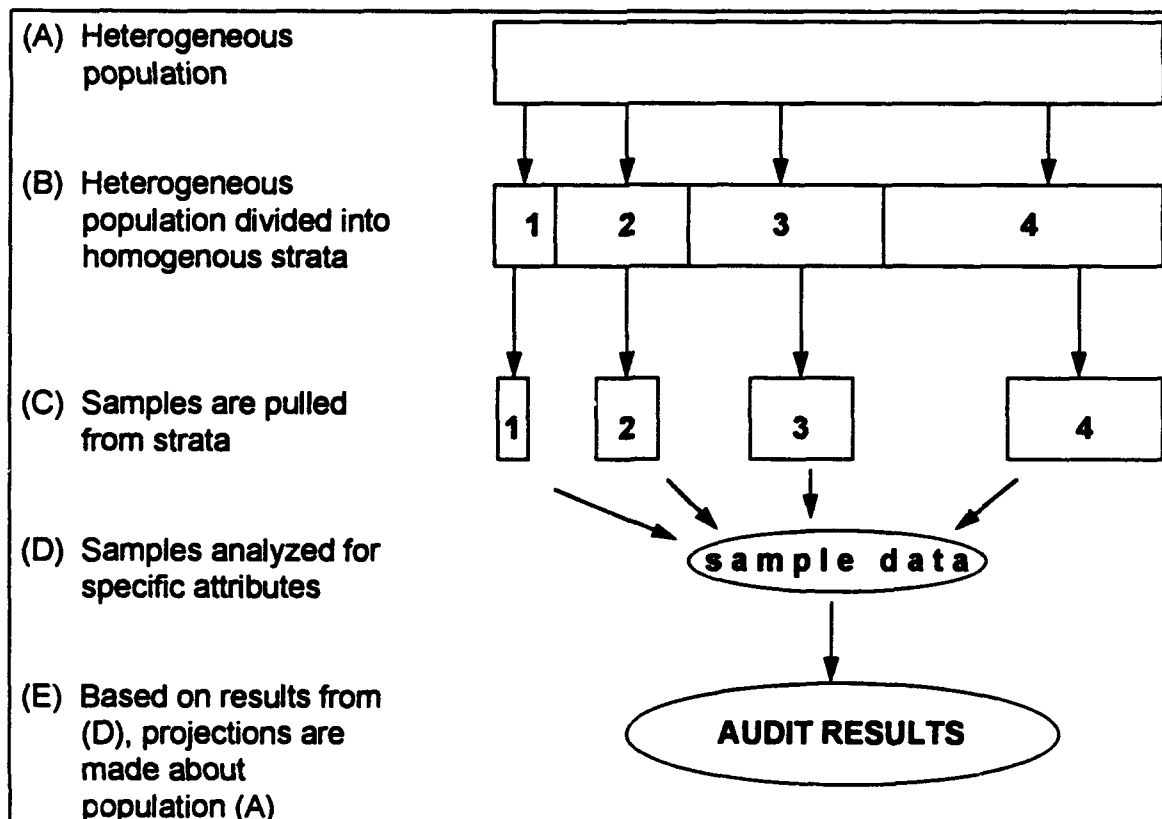


Figure 1. Stratified Sampling Process

4. Advantages of stratified sampling

There are three major advantages of using stratified sampling:

(1) **Efficiency.** Frequently a stratified sample can produce a satisfactory result with a minimum of effort and expense when compared to a simple random sample. One probability sampling procedure is more efficient than another if it offers, at a given level of confidence, the same precision at less cost or greater precision at the same cost. For example, a hypothetical sample of nine teachers could be observed to find their average years of education. In Table I their variance equals 2.61. However, when teachers are stratified by the level of school they teach and the two teachers with the greatest difference in years of education are sampled, the variance of each stratum significantly declines. These variances, which measure the variability of years of education within each stratum, are much smaller than the variance of the entire population. Consequently, in this instance, a stratified random sample of a given size will yield a substantially more precise estimate than a simple random sample of greater size, since much smaller variability is encountered within each stratum.

Table I. THE EFFECTS OF STRATIFICATION

Teacher	Level of School	Years of Education		Teacher	Level of School	Years of Education
1	Elementary	15.5		6	College	20.5
2	College	19.0		7	High School	17.5
3	High School	16.5		8	College	19.5
4	Elementary	16.0		9	Elementary	16.5
5	High School	18.0				
Variance = 2.61						
Stratum 1: Elementary		Stratum 2: High School		Stratum 3: College		
Teacher	Years of Education	Teacher	Years of Education	Teacher	Years of Education	
1	15.5	3	16.5	2	19.0	
9	16.5	5	18.0	6	20.5	
Variance = 0.25		Variance = 0.56		Variance = 0.56		
Source: <i>Applied Statistics</i> , Neter, 1993, p.735						

However, for stratified sampling to be efficient, the strata must be designed to contain relatively homogeneous elements. Homogeneity is accomplished when the basis of stratification is related to the characteristic under study (e.g., level of school taught and years of education). While the process of stratifying a sample may be considered an extra effort when compared to taking a simple random sample of equal total size, the stratification process normally allows for a *decrease* in the total sample size (see Table I). This decrease in total sample size is desirable because precision can be maintained, or improved, and the sampling time and costs are reduced.

(2) Information about subpopulations. Stratified sampling can provide secondary findings of strata characteristics in addition to the primary findings of overall population characteristics. For example, a study by a university of the effects of tuition increases may be primarily meant to provide the extent of sentiment from all students, but it also provides valuable separate information about graduate and undergraduate students, minority students, male and female students, etc. (Neter, 1993, p. 735).

(3) Feasibility. Sometimes stratified sampling is simply the most feasible. For example, if police records are computerized in a city and manual in the rest of the state, administrative consideration may require separate sampling in the city and elsewhere in the state (Neter, 1993, p. 735).

B. ESTIMATION SAMPLING

The function of estimation sampling for auditors is to determine two things. First, an attempt is made to determine the number of occurrences of some attribute, such as errors, violations, etc. in the population. Secondly, once the attributes are identified, it is usually appropriate to determine the magnitude of those attributes in order to make suggestions for improving the operation under review.

1. Frequency Estimation (Attributes)

Often it is sufficient simply to determine how many times a certain attribute occurs. Once this has been found it is only necessary to total the occurrences before making conclusions and judgments about the situation that was observed. This is particularly true when all attributes (errors, violations, etc.) carry equal weight and one is not more important than another.

During the DODIG audit, the "attributes" being checked were the occurrence of excessive (unnecessary or premature) purchases. Because each one of the excessive purchases observed had a different dollar value, they were not sampled equally. Instead, a higher percentage of the purchase requests in the high dollar value strata were selected.

2. Dollar Value Estimation (Variables)

Variables estimation is the statistical method used to estimate dollar values. The objective of this method is to estimate an average value of a group of items by means of a sample with an assurance equivalent to the confidence level that the sample average will be within a range of some specified amount from the true average which would have been attained if all items in the entire population had been averaged (Hill, 1962, p. 31).

The distinction between frequency and dollar value estimation can be seen when examining inventories. On the one hand, a company may want to check the reliability of inventory control. Items in the storeroom are identified and then compared to the inventory records to ensure that a stock record exists for every item. On the other hand, the company may want to determine the value of its inventory. A sample of the inventory would be counted, priced, and extended. On the basis of the average value of the items selected, the aggregate value of the inventory could be determined within the limits of reliability attained and compared to the company's figures (Hill, 1962, p. 15).

In the case of the DODIG and the ICPs, the sample purchase requests were analyzed and a determination was made as to whether or not they were excessive. Secondly, the DODIG determined from a relatively small sample of purchase requests the

dollar value of all excessive purchase requests. This method of dollar value estimation will be examined more closely in Chapters III and IV.

C. CONFIDENCE INTERVALS

In the DODIG report, the estimated cost savings are given with a 95 percent confidence level and a sampling precision of $\pm 3\%$ (Jones, February 1, 1993, p. 33). Therefore it is critical to know exactly what this means. Confidence intervals are used for interval estimation. Interval estimation is the quoting of bounds between which the true population mean lies. This is appropriate when it is desired to give some indication of the accuracy with which a parameter is estimated by the sample (Robinson, 1992, p. 120).

In any situation where there is less than a complete sample (e.g., less than the entire population), there will be a margin of uncertainty surrounding the results. This degree of uncertainty may be measured mathematically and expressed in terms of reliability (e.g., 95% confidence) and precision (e.g., $\pm 3\%$). Precision and reliability are mathematically interdependent and statistically inseparable. "The precision of an estimate indicates the range within which it is expected to be accurate, and the reliability (or confidence) is the probability of achieving this accuracy" (Newman, 1976, p. 30).

Probability statements cannot be made about a single event. For example, if a coin is flipped once, it cannot be said that the probability of the coin having turned up heads is one-half. The coin either turns up heads or it does not. If, however, the coin is flipped numerous times, a meaningful statement can be made that the relative frequency of heads turning up approaches 0.5 (Cyert, 1962, p. 8).

The reliability statements made about a confidence interval are similar to the statements on coin flipping. It is inaccurate to specify the probability that a particular confidence interval contains the true value being estimated. However, meaningful statements can be made about the probability that similar confidence intervals, in a series of sample estimates, contain the true value being estimated. For example, it might be

appropriate to make the statement that, in repeated trials, a confidence interval of plus or minus "x" dollars about a sample estimate will contain the true value being estimated 90% of the time (Cyert, 1962, p. 8).

An important question is what size confidence interval should be used? The answer depends on the tradeoff between the confidence level and the width of the interval. To obtain higher confidence from the same sample, one must be willing to accept a larger margin of error (wider interval). The way to attain higher confidence and still have a short interval is to take a larger sample. That means that, for a fixed level of confidence (e.g., 95%), the confidence interval (e.g., $\pm 3\%$) decreases as the sample size increases. Or, for a given interval, the confidence level increases as the sample size increases (Moore, 1979, p. 276).

D. STRATIFICATION

The final bottom line impact of this audit consisted of the values for three measures - the universe of contracts, the percent of unnecessary contracts, and the Due In Long Supply (DILS) rate. Each of these is further discussed in Chapter III.F. However, because the DILS rate is determined through the use of "stratification," this term, as it applies to the DILS rate, must first be clarified.

It is important to understand that we are talking about the inventory management definition of stratification (also called "STRAT") which is an entirely separate concept from the stratified sampling process described earlier. A complete description of stratification and all its related equations is beyond the scope of this thesis. However, it should be understood that stratification is mainly the semi-annual process of comparing various assets and requirements levels and placing each in a priority sequence. The

stratification program is used to analyze each inventory item. After all items have been stratified, the total stratification budget request is obtained by adding up the dollar value of future expected procurement/repair deficiencies during the Budget Year (FMSO 5230/52A, July 17, 1994, p. 1-1). Stratification is the basis for budget requests needed to operate the inventory system.

Because stratification tries to predict future needs, it uses three periods over which material usage must be considered: Current Year, Apportionment Year, and Budget Year. To initiate the forecast, "STRAT" first looks at the Opening Position (the "right now" inventory and requirements). The Opening Position is computed by taking the current system assets on hand and matching them against current system requirements such as safety levels and lead time demand. Next, demands and forecasted order receipts are computed for the Current Year, which covers the remainder of the fiscal year after the Opening Position date. The Apportionment Year includes the twelve months following the Current Year and its demands and forecasted order receipts are likewise estimated. Finally, the Budget Year's demands and required orders are determined. The Budget Year extends from the end of the Apportionment Year through to the following September 30th. When stratification matches assets against requirements, it may not compute material requirements but rather excess inventory and may therefore recommend review of long supply conditions, where on hand and due-in material exceeds material retention ceilings.

III. PRESENTATION OF DATA

A. INTRODUCTION

The objectives of the DODIG audit were to determine whether quantities of repairable items to be bought on outstanding procurements by the Navy wholesale ICPs were warranted by anticipated requirements and whether internal management controls over the determination of the procurement requirements were effective (Jones, February 1, 1993, p.1).

Within the last five years, the Department of Defense Inspector General has changed the type of inventory audits conducted in the services. Previously the audits concentrated primarily on items that were newly authorized to be placed in inventory. Items that were already being carried in stock were not included in the audit. Additionally, procurement lead times and customer demand were the main areas reviewed (Interview with Mr. J. Chaney, April 1994).

In 1990, DODIG decided to take a slightly different approach to their audit. At that time they began to look at all items with procurements in process, whether or not the procurements were for new items. Initially, the audits were to be DOD-wide. However, DODIG eventually chose to audit service by service and then made a further split between repairables and consumables.

As of September 30, 1990, the ICPs were in the process of procuring approximately \$863 million of stock for 11,308 depot level repairable (DLR) line items (see Column (2) in Table II below). The process to purchase these items starts when automated programs are triggered to reorder and the program recommends a purchase quantity to meet the stockage objective. The inventory manager then reviews the requirement and other relevant data to verify the accuracy of the computation. Approved purchase requests serve as authorization for the ICP's procurement organization to buy the material.

Table II. SELECTION OF SAMPLE UNIVERSE

(1)	(2)	(3)
	UNIVERSE	PURCHASE REQUESTS GREATER THAN \$100,000
ASO		889
SPCC		541
TOTAL LINE ITEMS	11,308	1430
		(12.6% of 11,308)
ASO		\$495,900,000
SPCC		\$195,300,000
TOTAL DOLLAR VALUE	\$863,000,000	\$691,200,000
		(80.1% of \$863M)

B. AUDIT SCOPE

DODIG initially intended to narrow its sample universe from 11,308 to only those procurements valued at \$100,000 or greater (Column (3) of Table II) independent of ICP. This cutoff of \$100,000 was chosen by DODIG because purchases of this value required greater supervisory review and checking the effectiveness of the review process was one purpose of the audit. At the same time, a great majority of the total dollar value of procurements in process were included above the \$100,000 level. Specifically, this decision narrowed the population to 1430 line items (13 percent of 11,308) but still included 80 percent of the total value of the procurements in process (\$691.2 million).

C. AUDIT SAMPLE

From the universe of 1430 line items, initially a sample of 107 line items was selected. The 107 line items totaled \$229.4 million in procurements (see Column (3) of Table III). Appendix B gives a detailed summary of all purchase requests greater than \$100,000. Before the sample was taken, the purchases were identified by ICP, and within ICPs they were further subdivided by size of procurement. The four procurement strata

were: (1) greater than \$5 million, (2) \$2.5 - 5 million, (3) \$1 - 2.49 million, and (4) \$100 - 999 thousand.

Table III. INITIAL SAMPLE SELECTION

(1)	(2)	(3)	(4)
	PURCHASE REQUESTS GREATER THAN \$100,000	SAMPLE FROM 1430 ITEMS	ITEMS NOT QUALIFIED (DISCARDED)
ASO	889	69	15
SPCC	541	38	6
TOTAL	1430	107	21
		(7.5% of 1430)	(19.6% of 107)
ASO	\$495,900,000	\$172,600,000	\$30,700,000
SPCC	\$195,300,000	\$56,800,000	\$10,400,000
TOTAL	\$691,200,000	\$229,400,000	\$41,100,000
		(33.2% of \$691.2M)	(17.9% of \$229.4M)

Of the 107 items (listed in Appendix C), 21 did not "meet the criteria" of the DODIG review. These 21 items (see Column (4) of Table III) were excluded from further review either because the purchases were not in process at the sample cutoff date (e.g., purchase requests were canceled or contracts had already been awarded) or because the purchases were for items that were managed using "consumable item" management techniques. It was necessary for audited items to be in process so that any unnecessary procurements identified by the audit could be canceled, if the Navy concurred with the findings. In reality, \$55.1 million in purchase requests were canceled by the ICPs during the audit. Of the \$55.1 million, \$33.0 million were initiated by the ICPs and the hardware systems commands (HSC), independent of the audit, and the remaining \$22.1 million were canceled in response to the audit (Jones, February 1, 1993, p. 25). Purchases that were excluded because they were treated as consumable items were included in a separate audit

entitled "Military Department Requirements for Currently Procured Wholesale Inventories for Consumable Items," DODIG Report No. 91-106 of June 28, 1991.

The \$41.1 million item adjustment shown in Column (3) of Table IV is segmented by ICP and stratum in Appendix D. This adjustment reduced the final audit sample to 86 line items involving purchases valued at \$188.3 million. This adjusted sample is shown in Column (4) of Table IV and is shown in detail in Appendix E.

Table IV. REVISED SAMPLE SELECTION

(1)	(2)	(3)	(4)
	SAMPLE FROM 1430 ITEMS	ITEMS NOT QUALIFIED (DISCARDED)	SAMPLED ITEMS FOR REVIEW
ASO	69	15	54
SPCC	38	6	32
TOTAL	107	21	86
			(80.4% OF 107)
ASO	\$172,600,000	\$30,700,000	\$141,900,000
SPCC	\$56,800,000	\$10,400,000	\$46,400,000
TOTAL	\$229,400,000	\$41,100,000	\$188,300,000
			(82.1% of \$229.4M)

Based on the previous adjustments, the Navy sample universe was adjusted to 1056 line items with purchases valued at \$520.2 million. The theory behind this process is discussed in Chapter IV.E. The adjustment of the sample universe was done in two parts - by line items and by dollar value. First, the number of line items of the new sample were compared to the old to get a percentage value. This value was then multiplied by the size of the old universe. This was done by stratum for each ICP and they were totaled to give the number of line items for the adjusted universe.

Table V. ASO LINE ITEM ADJUSTMENT

Stratum	Old Sample	New Sample	Old Universe	New Universe
1	8	7	8	7
2	20	15	26	20
3	17	15	66	58
4	24	17	789	559
	69	54	889	644

To show one example, Table V has been pulled from Appendix B and Appendix E. The calculation of the adjustment to ASO stratum 3 will be described. First, the sample size has changed from seventeen to fifteen. Fifteen divided by seventeen equals .88235. When .88235 is multiplied by the old universe size (66), the new universe size is obtained for ASO stratum 3; namely 58. This method holds true for each ICP's stratum and finally gives the "adjusted universe" size of 1056. Secondly, the adjustment to obtain the "new universe" value is calculated by getting the dollar size of the items discarded from the sample for each stratum, dividing it by the original sample value and then multiplying the quotient by the old universe value. As an example, in Table VI, stratum 3 is calculated by dividing \$2,601,156 by \$24,738,320. The quotient (.1051) is then multiplied by \$99,705,919 to get the estimated reduction in the universe value of \$10,483,762. This estimated reduction is labeled the "projected value" in Table VI as that is the term used by the DODIG. The final universe value is determined by doing this for each ICP's stratum to get the total value for the entire adjustment (items deleted). That value is \$170,972,701. When this adjustment value is subtracted from the original universe size, the adjusted universe value of \$520.2 million is obtained.

Table VI. ASO VALUE ADJUSTMENT

Stratum	Adjustment Value	Old Sample Value	Old Universe Value	Projected Value
1	\$9,006,800	\$78,092,994	\$78,092,994	\$9,006,800
2	17,394,859	64,160,301	83,865,837	22,737,337
3	2,601,156	24,738,320	99,705,919	10,483,762
4	1,717,478	5,648,744	237,191,420	72,117,091
	\$30,720,293	\$172,640,359	\$498,856,170	\$114,344,990

D. DODIG AUDIT

DODIG auditors examined the 86 purchase request documents related to the purchases in process as of September 30, 1990 to evaluate the basis for procurement decisions. They evaluated requirements data that were in effect at the time of the audit to determine whether requirements supported continuation of the procurement.

Specifically, the auditors (1) determined whether requirements forecasts were reasonable, (2) reviewed the accuracy of the forecasted demand rates, (3) evaluated the propriety of nondemand-based (additive) requirements, and (4) verified the accuracy of on-hand asset and due-in asset balances. Additionally, the auditors selectively reviewed other requirements data and other factors that affected the requirements forecast, such as past and future program data, survival and wearout rates, and repair cycle times (Jones, February 1, 1993, p. 2). The auditors did not merely consider whether the purchases were justified at the time of reorder, but also whether the initial order quantity was still justified at the time of the audit in September 1990. Any demand changes after the initiation of the audit were not considered part of the audit results. Any action, or inaction, on the part of inventory managers was not included in the audit results (Interview with Mr. J. Chaney, May 1994).

E. DATA CATEGORIZATION

All data presented in this section ultimately lead to the DODIG's calculation of projected monetary benefits (from the canceling of outstanding orders) derived from this audit. After all calculations and negotiations, this was determined to be \$68.2 million. Reading Table VII from left to right shows the progression to reach the projected monetary benefits (listed as "PMB"). First, the universe is divided into three categories. Second, the excessive purchase requests are pulled from the universe and divided into

three categories. Third, unnecessary purchase requests are pulled from the excessive purchase requests and divided into two categories. Finally, the purchase requests with a "potential monetary benefit" are pulled from the unnecessary purchases to determine the findings of the audit. The "other" category represents offsetting costs that reduce net potential benefits (Jones, February 1, 1993, p. 47). Each one of the eight groupings, except "Other," is broken down by ICP and stratum in Appendices G through M.

Table VII. SUMMARY OF AUDIT RESULTS

(1)		(2)		(3)	
Adjusted Universe		Excessive		Unnecessary	
Undetermin.	\$69,543,164	Non-projectable	\$46,046,289	PMB	\$68,245,887
Reasonable	297,062,229	Premature	32,123,629	Other	10,507,569
Excessive	156,563,299	Unnecessary	78,753,456		
	\$523,168,692		\$156,923,374		\$78,753,456

The first step after the dollar value of the adjusted universe was determined was to divide this universe dollar value into three non-overlapping categories - undeterminable, reasonable, and excessive. The total combined dollar value of these three categories must equal the dollar value of the entire universe (\$523.2 million). This total differs from the previous total value for "adjusted universe" (\$520.2 million) described in Section C and the reason for the difference will be discussed in Chapter IV.E.

1. Undeterminable

The first category into which the universe was subdivided was called "undeterminable." Undeterminable included all purchase requests for which DODIG was unable to determine the reasonableness of purchases because the ICPs could not provide verifiable requirements data as of September 30, 1990 or the requirements were dependent

on a management decision by an HSC (Jones, February 1, 1993, p. 34). Purchase requests were considered "excessive" if the quantity ordered exceeded the DODIG-defined "stockage objective" by more than twelve months of forecast requirements. Purchase requests that did not fit into either of these categories were considered "reasonable."

The method to make the projection based on the sample is identical for each of these three categories. First, the quantity (hits) and dollar values (hit values) are determined. A purchase request is considered a "hit" if it either wholly or partially fits into one of the three categories. For instance, one purchase request may be partially reasonable and partially excess. However, the dollar value of the two groupings cannot exceed the original purchase request value. For instance, a purchase request for \$110,000 may be determined to be \$10,000 excess by the auditors. This one line item would then fit the "reasonable" category with a "hit" value of \$100,000 and the "excessive" category with a "hit" value of \$10,000. Although the term "hit" implies a problem, the DODIG uses the term simply to classify purchase requests and their dollar values.

Table VIII. ASO UNDETERMINABLE LINE ITEM PROJECTION

Stratum	Hits	Sample Quantity	Universe Quantity	Projected Hits
1	2	8	8	2
2	4	20	26	5
3	3	17	66	12
4	2	24	789	66
	11	69	889	85

To make the projection, the hit quantity is divided by the sample quantity and that quotient is then multiplied by the universe quantity. The answer is then rounded to the nearest integer. Excerpts from Appendices B and G are given as examples. In Table

VIII, ASO stratum 3 shows three undeterminable items. Three is then divided by the sample quantity of seventeen. The quotient (.17647) is then multiplied by the universe quantity of 66 to get twelve projected hits for stratum 3. The same methodology is used to get the projected values in Table IX. Specifically, \$4,646,285 divided by \$24,738,320 equals .187817321. Then the universe value of \$99,705,919 multiplied by .187817321 equals \$18,726,500 (the projected value for ASO stratum 3).

Table IX. ASO UNDETERMINABLE VALUE PROJECTION

(1)	(2)	(3)	(4)	(5)
Stratum	Hit Value	Sample Value	Universe Value	Projected Value
1	\$18,015,238	\$78,092,994	\$78,092,994	\$18,015,238
2	12,573,801	64,160,301	83,865,837	16,435,589
3	4,646,285	24,738,320	99,705,919	18,726,500
4	389,754	5,648,744	237,191,420	16,365,837
	\$35,625,078	\$172,640,358	\$498,856,170	\$69,543,164

A summary of the projected hits and projected values for undeterminable, reasonable, and excessive purchase requests is shown in Appendices G through I. The total projected values of these three categories does equal \$523,168,692 as shown in Table VII.

2. Excessive

The next major step the DODIG took to determine projected monetary benefits was to break down the "excessive" category into three subcategories. The DODIG calls these groups non-projectable, premature, and unnecessary. The total of these three areas should equal \$156,563,299, which is the total for the "excessive" category as shown in Appendix I and Column (1) of Table VII. However, a thorough review of all working papers shows that these three actually total \$156,923,374 - a difference of \$360,075

(Barton, Nov. 16, 1992, summary page). The impact of this difference will be discussed in Chapter IV.E.

The method used to project the number of hits and the hit value in the universe was the same as that used to make projections for the adjusted universe described in the previous section. The number of hits in the sample stratum are divided by the stratum sample size and the quotient is multiplied by the universe size. The product is then rounded to the nearest whole number. This is then done for each stratum and the results are totaled for each ICP to get the projected hits and projected total hit value.

3. Unnecessary

The final category that the DODIG calculated was the "projected monetary benefit" portion of the purchase requests determined to be unnecessary. The DODIG projected \$68.2 million of potential monetary benefits from the unnecessary items (see Table VII). The \$10.6 million difference (\$78.8 million total unnecessary minus \$68.2 million) represents offsetting costs to repair unserviceable assets (Jones, February 1, 1993, p. 47). However, the audit report does not explain the specific nature of those costs or how or why they offset benefits from avoiding unnecessary purchases. The projection of the potential monetary benefits can be seen in Appendix M. The methodology used to make the projection is also identical to the calculations used to make projections for the adjusted universe as described above in section E.1.

F. AUDIT IMPACT

The actual final impact of this audit was reported in an NC-2 Report submitted by the Naval Supply Systems Command (NAVSUP) with the reapportionment budget to the

Navy Comptroller (NAVCOMPT). This report summarized all audit findings and recommendations from the previous fiscal year. The purpose of this report was to ensure that monetary benefit claims (budget reduction) are actually incorporated into the following year's budget.

The bottom line for this particular DODIG audit was calculated using three measures:

- (1) U (universe) - \$520.1 million. The value of the adjusted universe size reported in the audit.
- (2) C (unnecessary contracts) - 71%. The audit projected \$110.9 million in premature or unnecessary procurements (see Table VII). Unnecessary contracts were projected to total \$78.8 million (or 71% of \$110.9 million).
- (3) D (DILS) - 8.23%. The Due In Long Supply (DILS) rate calculated during each semiannual STRAT (as described in Chapter II.D). NAVSUP figured this DILS rate by estimating the total value of all contracts in excess of known requirements during the Budget Year and dividing this figure by the total value of contracts expected to be on order during the Budget Year. The average DILS rate from September 1990 through September 1991 was 8.23%.

To obtain the figure that NAVSUP used to report to NAVCOMPT, the following equation was used: $U \times C \times D = \text{Agreed Potential Monetary Benefit}$. NAVSUP reported this agreed potential monetary benefit to be \$30.4 million or $\$520.1\text{M} \times 71\% \times 8.23\%$. This figure was used by NAVSUP in their arbitration hearing with the DODIG. The arbitrator was a representative from the Office of the Undersecretary of Defense for

Logistics (OUSD(L)). OUSD(L) determined that \$30.4 million was acceptable for NAVSUP to report to NAVCOMPT. Furthermore, because the \$30.4 million was classified as a one time "cost avoidance" for fiscal year 1991 rather than "savings" from the audit, NAVSUP was not required to incorporate this figure into their budget estimate for fiscal year 1994 but rather simply to list it as a recognized audit finding. In other words, a cost would have occurred if the audit had not taken place. However, because there was an audit, the costs are said to have never occurred.

IV. DATA ANALYSIS AND INTERPRETATION

A. INTRODUCTION

This chapter includes a critique of the entire process used by the DODIG to make conclusions found in the audit report. Specifically, Section B discusses the DODIG selection of the universe size and sample size, strata, confidence level, and precision. This information was obtained from their audit report, the audit working papers provided by the Quantitative Methods Division of the DODIG, and discussion with the statisticians assigned to the audit. Section C begins the analysis of the DODIG decisions and then presents an alternative method of selecting the sample size along with two ways of dividing the selected sample over the chosen strata. Section D provides a method to test stated levels of confidence and precision. The chapter concludes with a discussion of several inconsistencies in the reported figures used during the audit.

B. DODIG STATISTICAL DECISIONS

1. Adjusted Universe

As mentioned in Chapter III.C, the DODIG treated items that they did not want to include in the audit in the same way as the attributes they did want to include. For example, when their samples uncovered excessive or reasonable purchase requests, projections were used to calculate the number and value of these purchase requests in the universe. At the same time, when the DODIG found sampled purchase requests that they determined were outside the scope of the audit, they used the same projection procedures to "adjust" the universe. While the process would have been time consuming, the only

statistically valid way to "adjust" the universe is by going through the original universe and discarding the purchase requests deemed to be outside the scope of the audit. Only after this is complete can a true sample be used to make predictions about the population of relevant purchase requests.

Specifically, the initial sample of 107 items found 21 items that were determined to be outside the scope of the audit. Based on these 21 items, the DODIG determined it likely that there was a similar proportion of items in the entire population of 1430. The projection calculated 374 total population items outside the audit and therefore the population was "adjusted" to 1056 items. A similar projection found \$171 million of the original population value to be outside the audit's scope. Therefore, the population value was reduced from \$691.1 million to \$520.1 million. Both of these projections were made the same way the projections were made for every other grouping used during the audit. This procedure is described in Chapter III.E. The results of this projection are then shown in Appendix D. Again, the only truly valid way to adjust the population is to go item-by-item through the population and eliminate items outside the audit's scope. Only when this process is complete can a usable sample be selected.

However, the DODIG did not actually use the "new" universe of 1056 items valued at \$520.1 million. Instead the original universe size and value were used throughout the entire audit and in all calculations. The DODIG's actual purpose of determining the "adjustment" was to ensure the total value of those items which were outside the audit's scope was set aside and not mingled with any other part of the audit.

Therefore, the audit results did not become flawed as they may have if the "adjusted universe value" was actually used in calculations.

2. Sample Size

As previously discussed, the chosen sample size for this audit was 107 out of 1430 items. Despite the fact that this audit of the Navy commands was similar to audits the DODIG conducted with the Air Force and Army, there were no sample size selection procedures available for review. Once the decision to sample 107 items was made, there was no algorithm used to divide the 107 into the four strata (Interview with Dr. D. Barton, May 1994). Instead, conscious decisions were made to make 100% reviews of the top dollar value stratum, sample approximately 80% of the second stratum, and then divide the remaining items over the bottom two strata. Once the decision was made to sample "x" percent from each stratum, the purchase requests were chosen by using simple random sampling.

3. Stratification

One of the auditors' first decisions after deciding to sample only those purchase requests valued at \$100,000 or larger was to stratify the universe of purchase requests into four strata. As mentioned in Section B.2, the stated purpose of the strata sizing decisions was to ensure a review of a high percentage of the highest dollar value purchase requests. In other words, the top strata should not include too many items. The strata breaks were based partially on "natural breaks" in the universe of purchase requests and partially on the level of supervisory review required for the purchase requests (Interview with Mr. J. Chaney, May 1994).

One method to verify the effectiveness of stratification is to compare the standard deviation of the samples before and after stratification. Secondly, the coefficient of variation should be determined to see the relative variability. Equation (1) in Appendix A was used to make that calculation. As can be seen in Table X, simply dividing the sample between ASO and SPCC does not decrease the relative variability - SPCC's coefficient of variation (136.74) is higher than the combined coefficient of variation (128.71). The true advantage of stratification is apparent when the sample is broken into strata by the size of the purchase request. This can be seen by the decrease in the coefficients of variation once the stratifications are made.

Table X. SUMMARY OF STANDARD DEVIATIONS OF PURCHASE REQUEST VALUES AND COEFFICIENTS OF VARIATION

	Standard Deviation (\$)	Coefficient of Variation (Pct)
ASO and SPCC	2,759,542.57	128.71
ASO	3,038,801.45	121.45
ASO - Stratum 1	2,494,853.84	25.56
ASO - Stratum 2	655,963.22	20.45
ASO - Stratum 3	378,072.98	25.98
ASO - Stratum 4	128,973.78	54.80
SPCC	2,042,441.49	136.74
SPCC - Stratum 1	2,525,926.84	30.07
SPCC - Stratum 2	420,372.86	13.02
SPCC - Stratum 3	503,727.75	32.51
SPCC - Stratum 4	172,256.03	67.72

4. Confidence Level and Precision

An important statement in the report of audit findings was that "the sample results were projected with a 95% confidence level and a sampling precision of ± 3 percent for dollars" (Jones, February 1, 1993, p. 33). In terms of the audit results, this translates to a \$68,245,887 finding with a 95 percent confidence and a precision of less

than $\pm \$2,047,377$. In the audit working papers the "margin of error" is stated as \$552,396. Clearly, this is within the stated precision. However, these figures are analyzed in Section D to test their accuracy.

C. SAMPLE SIZE ANALYSIS

1. Precision

Before attempting to verify the precision of the audit results, equations (2) through (4) in Appendix A can be used to obtain a confidence interval for the projected number of hits made by the DODIG (seen in Appendix M). First, it is necessary to obtain the population proportion. Secondly, an estimated variance should be determined. Then the variance can be used to obtain a confidence interval at the 95% confidence level. The step-by-step process is shown below.

a. Estimator of the Population Proportion

In order to estimate the proportion (fraction) of purchase requests that were unnecessary, with projected monetary benefits, the DODIG auditors used equation (2) in Appendix A. The population was divided into strata and a sample was taken from each stratum. The purchase requests were then thoroughly analyzed to see which were considered unnecessary with projected monetary benefits. The following calculations show how DODIG obtained the projected quantities for ASO and SPCC, respectively. These figures match those seen in Appendix M.

First, the formula for equation (2) is:

$$\hat{p}_u = \frac{1}{N} \sum_{i=1}^L N_i \hat{p}_i ,$$

where N is the total universe size, N_i is the universe size by stratum as shown in Appendix B, and \hat{p}_i is the proportion of the sample that was found to be unnecessary with a projected monetary benefit. Next, values of these parameters were substituted into the equation.

$$\begin{aligned}\text{ASO: } \hat{p}_u &= \frac{1}{889} [8(0) + 26(.15) + 66(.1765) + 789(.25)] \\ &= \frac{1}{889} [212.797] \\ &= .2394\end{aligned}$$

$$\begin{aligned}\text{SPCC: } \hat{p}_u &= \frac{1}{541} [2(0) + 6(.1667) + 21(.3) + 512(.35)] \\ &= \frac{1}{541} [186.5502] \\ &= .3447\end{aligned}$$

For example, $\hat{p}_2 = .15$ (above) for ASO and was the ratio 3/20 where 3 was the number of hits for ASO stratum 2 and 20 was the sample size for ASO stratum 2. The \hat{p}_i values were computed from the number of hits in Appendix M and the stratum sample sizes in Appendix B.

The .2394 and .3447 figures are used in Subsection c to determine the bounds on the errors of estimation. Also, the summations of $N_i \hat{p}_i$ (212.797 and 186.55 for ASO and SPCC, respectively) show how the DODIG projected the number of unnecessary purchase requests, listed in Appendix M.

b. Estimated Variance

The estimated variance can be obtained by using equation (3) from Appendix

A. The estimated variance is needed to obtain the confidence interval shown in Subsection c below.

$$\hat{V}(\hat{p}_n) = \frac{1}{N^2} \sum_{i=1}^L N_i^2 \left(\frac{N_i - n_i}{N_i} \right) \left(\frac{\hat{p}_i \hat{q}_i}{n_i - 1} \right)$$

Calculating this solution is simplified by separately calculating $\left(\frac{N_i - n_i}{N_i} \right) \left(\frac{\hat{p}_i \hat{q}_i}{n_i - 1} \right)$, which is also equal to $\hat{V}(\hat{p}_i)$. For ASO, $\hat{V}(\hat{p}_i)$ is determined as follows:

$$\hat{V}(\hat{p}_1) = 0;$$

$$\hat{V}(\hat{p}_2) = \left(\frac{26 - 20}{26} \right) \left[\frac{(.85)(.15)}{19} \right] = .001548583;$$

$$\hat{V}(\hat{p}_3) = \left(\frac{66 - 17}{66} \right) \left[\frac{(.1765)(.8235)}{16} \right] = .006744356;$$

$$\hat{V}(\hat{p}_4) = \left(\frac{789 - 24}{789} \right) \left[\frac{(.25)(.75)}{23} \right] = .007904199;$$

Substituting these values into equation (3) of Appendix A gives:

$$\begin{aligned} \hat{V}(\hat{p}_n) &= \frac{1}{(889)^2} [0 + (26)^2 (.00155) + (66)^2 (.00674) + (789)^2 (.0079)] \\ &= \frac{4950.955}{(889)^2} = .006264486. \end{aligned}$$

Similarly for SPCC:

$$\hat{V}(\hat{p}_1) = 0;$$

$$\hat{V}(\hat{p}_2) = 0;$$

$$\hat{V}(\hat{p}_3) = \left(\frac{21-10}{21} \right) \left[\frac{(.3)(.7)}{9} \right] = .01222;$$

$$\hat{V}(\hat{p}_4) = \left(\frac{512-20}{512} \right) \left[\frac{(.35)(.65)}{19} \right] = .011505962.$$

and, therefore

$$\begin{aligned} \hat{V}(\hat{p}_x) &= \frac{1}{(541)^2} [0 + 0 + (21)^2(.012) + (512)^2(.011506)] \\ &= \frac{3021.608}{(541)^2} = .0103239. \end{aligned}$$

c. Bound on Error of Estimation

The results obtained from Subsections a and b are then used with equation (4) from Appendix A to obtain a 95% confidence level. For ASO, the 95% confidence interval for the fraction of purchase requests having projected monetary benefits is given by

$$\hat{p}_x \pm 2\sqrt{\hat{V}(\hat{p}_x)} = .2394 \pm 2\sqrt{.006264486}$$

$$.2394 \pm .158297016$$

$$\text{or } .0811 \leq \hat{p}_x \leq .3977$$

Similarly, for SPCC the 95% confidence interval is

$$.3447 \pm 2\sqrt{.010323899}$$

$$.3447 \pm .203213174$$

$$\text{or } .1415 \leq \hat{p}_x \leq .5479$$

From Section 2 and Appendix M, the estimated number of unnecessary purchase requests for ASO and SPCC were 213 and 186, respectively. Using the results from this section, the 95% confidence interval for ASO would be from 72 ($889 \times .0811$) to

354 ($889 \times .3977$). For SPCC, the confidence interval is from 77 to 296. The size of these intervals is quite large. "We could reduce this bound and make the estimator more precise by increasing the sample size" (Mendenhall, 1990, p. 118).

In an effort to get a tighter bound on the errors of estimation, the strata for ASO and SPCC were combined and the calculations were again made to determine a 95% confidence interval. The aggregate variance decreased to .00388 from .00626 (for ASO) and .01032 (for SPCC). However, the range was still from 230 ($1430 \times .161$) to 586 ($1430 \times .41$). The range is less than it is for ASO and SPCC calculated separately. However, the bounds on the number of estimated unnecessary purchase requests would still be considered large.

2. Selecting Sample Size and Stratification

The previous section looked at how much precision existed for the projected number of hits found by the DODIG. This section considers methods to increase the precision of sample hits along with guidelines to subdivide the chosen sample size among the given strata. The two options are based on allocating the sample size among the strata in proportion to the universe size and then in proportion to the universe value.

a. Sample Size Selection

In order to ensure increased reliability from sampling, methods have been used to select sample sizes based on (1) the expected rate of occurrence of the specified characteristic, (2) the desired confidence level, (3) the number of items in the universe, and (4) the desired reliability. Appendix N was taken from (Hill, 1962) and has taken into consideration each of these factors. Its values are based on numerous calculations of

equation (5) in Appendix A. When the rate of occurrence is expected to be above 10%, the table with a 50% rate of occurrence is used to select a sample size. In this audit, the projected rate of occurrence, as given in Appendix M, turned out to be 28% (399 + 1430). Finally, the sample size would have been 624 if the auditors were only looking at sample attributes and not sample values.

b. Proportional Sample Sizes

This section and section c assume that the sample size of 107 is correct and proposes two processes to spread the 107 over the four strata. In actuality, the sample size used by the DODIG was not proven correct. But, because the procedure to choose the 107 items was not available for review, this sample size will be the one allocated over the four strata.

Table XI. THE EFFECT OF PROPORTIONAL SAMPLE SIZES

(1) Stratum	(2) Universe Size	(3) Percent of Universe	(4) Sample Size	(5) Percent of Sample	(6) Adjusted Sample
ASO - 1	8	0.9	8	11.6	1
2	26	2.9	20	29.0	2
3	66	7.4	17	24.6	5
4	789	88.8	24	34.8	61
	889	100.0	69	100.0	69
SPCC - 1	2	0.4	2	5.3	1
2	6	1.1	6	15.8	1
3	21	3.9	10	26.3	1
4	512	24.6	20	52.6	35
	541	100.0	38	100.0	38

One possibility is to spread the sample over the strata in proportion to the universe size (Cochran, 1963, p. 89). Table XI shows what percentage of the universe was in each stratum and then what percentage of the sample was in each stratum. Column

(3) shows that for both ASO and SPCC approximately 90% of the universe items are in the low dollar value category. Column (4) shows the sample sizes initially selected for each stratum. Column (5) shows the percentage that each stratum sample was of the whole for each ICP. If the sample size had stayed at 107 and was split between ASO and SPCC as in the audit and the sample had been spread over the strata in the same percentages as the four strata were of the universe, then the sample sizes would have been the spread seen in Column (6). However, this sample stratification would not have met the stated purpose of capturing a high percentage of the universe's value with the smallest possible sample.

c. Proportional Sample Values

To reflect the goal of sampling a high percentage of the dollar value of the universe, the sample per stratum could have been determined by using the percentage of the universe value contained within each stratum (see Column (3) of Table XII). This method would avoid results like those currently seen in the fourth strata, where approximately 50% of the universe value is included but less than 10% of the sample value. When Column (3) is used to determine the appropriate sample size within each strata, the adjusted sample size (Column (6)) is not nearly as skewed to include the high dollar value strata as the actual sample itself (Column (7)). However, the top dollar value stratum sample sizes do not change for either ASO or SPCC. Instead, the two middle strata have smaller samples and the lowest dollar value strata sample sizes increase. Column (6) is calculated by multiplying the column (3) percentage by the original ICP total sample size. For example, ASO stratum 3 is calculated by multiplying 20.1% by 69

to obtain 14. When the product exceeds the entire stratum population as in SPCC stratum 1 ($8.6\% \times 38 = 3$), then the difference (1) is added to the next highest stratum. This decision ensures the emphasis remains on the highest dollar value strata, as in the DODIG sampling plan. While this sample distribution is closer to the actual sample than that seen in Section b, it still emphasizes the fourth stratum more than the actual did.

Table XII. THE EFFECT OF PROPORTIONAL SAMPLE VALUES

(1) Stratum	(2) Universe	(3) Percent of Universe	(4) Sample	(5) Percent of Sample	(6) Adjusted Sample	(7) Actual Sample
ASO - 1	\$78,092,994	15.8	\$78,092,994	45.2	8	8
2	83,895,837	16.9	64,160,301	37.2	14	20
3	93,705,919	20.1	24,731,320	14.0	14	17
4	234,191,420	47.2	5,648,744	3.3	33	24
	495,886,170	100.0	172,640,359	100.0	69	69
SPCC - 1	16,801,000	8.6	16,801,000	29.6	2	2
2	19,377,237	9.9	19,377,237	34.1	5	6
3	34,665,847	17.8	15,493,707	27.3	7	10
4	124,441,089	63.7	5,087,281	2.9	24	20
	195,285,173	100.0	56,759,225	100.0	38	38

D. CONFIDENCE LEVEL AND PRECISION ANALYSIS

When statements are made about a sample result's confidence level and precision, it is important to be able to test these measures. Using a procedure used by Des Raj in *The Design of Sample Surveys*, this section does exactly that. Equations (1), and (6) through (9), in Appendix A are used here to determine the precision of the stated results, given the stated 95% confidence level. Both SPCC and ASO are combined to ensure that the tightest possible interval is obtained.

The first step is to gather the data shown in Table XIII. Columns (2) and (3) of Table XIII are taken from Appendix B. Column (4) can be calculated by dividing the total stratum hit value from Appendix M by the sample size (Column(3)). For example, stratum

2 (\$210,006) is calculated by dividing \$5,460,151 by 26. Column (5) is calculated using equation (6) from Appendix A. When Table XIII is complete, the population's total hit value is estimated. Using equation (7) from Appendix A:

$$\sum_{i=1}^L N_i \bar{y}_i = 10(0) + 32(210,006) + 87(209,250) + 1301(32,441)$$

$$= \$67,130,830$$

Table XIII. DATA TO DETERMINE PRECISION

(1)	(2)	(3)	(4)	(5)
Stratum	Universe (N_i)	Sample (n_i)	Hit Mean (\bar{y}_i)	Hit Variance (s_i^2)
1	10	10	0	0
2	32	26	\$210,006	\$352,532,666,060
3	87	27	209,250	207,296,123,200
4	1301	44	32,441	6,390,491,616
	1430	107		

This figure is less than the DODIG figure of \$68,245,847 shown in Appendix M. Secondly, equation (8) gives the mean value per purchase request of \$46,945. Third, equation (9) gives the following:

$$s^2(\hat{x}) = \sum_{i=1}^L N_i^2 \left(\frac{1}{n_i} \right) \left(1 - \frac{n_i}{N_i} \right) s_i^2$$

$$= 2,603,318,149,504 + 40,077,250,485,280 + 237,516,709,061,100$$

$$= \$280,197,277,695,884$$

When this result is divided by the square of the population (1430), the variance of the estimate is determined to be \$137,022,484 and the standard deviation \$11,706.

Using the procedures outlined in *The Design of Sample Surveys*, the next step is to determine the coefficient of variation using equation (1) in Appendix A:

$$\begin{aligned}
c &= 100 \left(\frac{s}{\bar{x}} \right) \\
&= 100 \left(\frac{11,706}{46,945} \right) \\
&= 24.935033\%
\end{aligned}$$

The coefficient of variation multiplied by the estimated total hit value of \$67,130,830 equals \$16,739,094. This figure is then used in equation (10) of Appendix A:

$$\begin{aligned}
s\{\bar{x}\} &= \frac{s}{\sqrt{n}} \\
&= \frac{16,739,094}{\sqrt{107}} \\
&= \$1,618,229
\end{aligned}$$

Then, multiplying this figure by the 95% confidence level factor of 1.96 gives \$3,171,729. Lastly, we divide \$3,171,729 by the estimated total hit value \$67,130,830 and find an actual precision of $\pm 4.7247\%$.

While 4.7247% may seem only slightly higher than 3%, in actuality 4.7247 is 57.5% greater than 3%. Additionally, the range allowed by the auditors was from \$66,198,510 to \$70,293,264. However, the range calculated here was from \$63,959,100 to \$70,302,560. This range is over \$2,000,000 greater than that claimed by the DODIG in the audit report.

E. DIFFERENCES

1. Universe of \$100,000 purchase requests

While reviewing the background information for the data that was published in the final audit report, several discrepancies arose in the supporting data. The first category that differs between the report and all supporting working papers is the dollar value of the universe of items which include purchase requests of \$100,000 or more. While there was an initial figure for the entire universe value (\$691.1 million), each of the four individual values was obtained through projections from the sample of 107 items. Although the four sample values were regularly being checked, re-checked, and changed during the audit, the four figures listed in Table XIV were apparently never re-totaled after the projections were made to verify that they matched the original universe value. All four are listed in the working paper summary with the dollar figures seen in Table XIV. This \$3 million difference is not significant but its impact carries forward to other categories, as seen in the next section.

Table XIV. \$100,000 UNIVERSE BREAKDOWN

	Value
Adjusted	\$170,972,701
Undeterminable	69,543,164
Excessive	156,563,299
Reasonable	<u>297,062,229</u>
Total	\$694,141,393

2. Adjusted Universe

The second major difference discovered was the total dollar value of the adjusted universe. In all the major conclusions, there were two different dollar sizes for the

adjusted universe: \$520.2 million, and \$523.2 million. This difference in adjusted universe value (\$3 million) is derived from figures taken from the report. As seen in Table XIV, the universe value was \$694,141,393. When the adjustment (\$170,972,701) is subtracted, the adjusted universe value of \$523,168,692 is obtained (see Column (1) of Table VII). Since the adjusted universe was not actually used in the audit estimate, this difference is of no practical significance.

3. ASO Universe

The next notable difference was the total dollar value for the 889 ASO purchase requests over \$100,000. When projections were made for the three largest categories (undeterminable, reasonable, and excessive), the working paper cover sheets correctly listed the ASO total as \$495,886,170 (see Appendix B), however the detailed working papers, used to calculate the projections, totaled \$498,856,171 (see column (4) of Table IX).

Table XV. ADJUSTED UNIVERSE CORRECTIONS

	Original Value	Corrected Value
Undeterminable	\$69,543,164	\$69,342,027
Reasonable	297,062,229	295,812,193
Excessive	<u>156,563,299</u>	<u>155,948,459</u>
	\$523,168,692	\$521,102,679

Table XVI. EXCESSIVE CORRECTIONS

	Original Value	Corrected Value
Non-projectable	\$46,046,289	\$45,905,070
Premature	32,123,629	31,941,125
Unnecessary	<u>78,753,456</u>	<u>78,462,046</u>
	\$156,923,374	\$156,308,241

Table XVII. UNNECESSARY CORRECTIONS

	Original Value	Corrected Value	Difference
PMB	\$68,245,887	\$67,987,400	
Other	10,507,569	10,474,646	
	\$78,753,456	\$78,462,046	

The specific mistakes were a \$30,000 shortage for ASO stratum 2 and a \$3,000,000 overage for ASO stratum 4. Tables XV through XVII give the recomputed projections for the categories originally listed in Table VII, Summary of Audit Results. While the original errors appeared significant (a \$2.97 million total difference), the true bottom line impact to be investigated is the change in the projected monetary benefits. As Table XVII shows, this amounted to \$258,487 which is less than the average value of one purchase request from the original universe ($\$691,171,343 \div 1430 = \$483,337$) and only .38% ($258,487 \div 68,245,887$) of the original projected monetary benefits.

4. Excessive

The final notable difference that merits attention is the total value of excessive purchase requests. As Table VII shows, when the total for the adjusted universe is calculated, the dollar value for excessive purchase requests is \$156,563,299. But when the components of the excessive category are shown, they total \$156,923,374. - a \$360,075 difference.

The best explanation for this difference is similar to that described in Section E.1 of this chapter. Again, it appears to be an instance where one set of figures was used to make the projection of the excessive value. Then, during a separate point in the audit, the three components of the excessive value were projected. Apparently, the three

components were never totaled to ensure they summed to the separately projected excessive total.

5. Summary

During the review of all figures behind the audit report, it became clear that the figures eventually were obtained in pieces. Retracing the process shows that the conclusions obtained were based on shifting foundations. The most important figures are those used to make projections. As seen in Section E.3, \$3 million and \$30 thousand errors are the basis for completely recalculating the projected monetary benefits of the entire audit. While the ultimate result may not be too much different, these corrections create a basis for questioning all assumptions made during the audit.

V. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

A. SUMMARY

The objective of this thesis was to examine how the Department of Defense Inspector General (DODIG) incorporated stratified sampling when auditing inventory control points. The primary focus was on how the DODIG used stratified sampling to project the potential monetary benefits of their findings. A secondary issue was to examine the stated confidence level and precision of their conclusions to see if they were as tight as stated in the final report.

Chapter II introduced the methodology for using stratified sampling to make projections accurately. For stratified sampling to be beneficial to the auditors, strata should be more homogeneous than the entire sample in order to reduce variability and reduce the total required sample size. Chapter II distinguished between frequency estimation and dollar value estimation, both of which were used during the DODIG audit. The chapter also included a brief discussion of confidence intervals and their use in stating conclusions about a sample mean. The final section of Chapter II was a description of the term "stratification" in inventory management. Stratification was critical for NAVSUP when reporting the impact of the audit. NAVSUP used three figures when calculating the audit's impact. One of the figures, the DILS rate, was determined from reviewing the "STRAT" while the audit was in process.

The data that was collected and used throughout the audit was described in Chapter III. Appendices B through M include the detailed data that was described in Chapter III. The beginning of Chapter III included a description of how the auditors

began the audit by narrowing the scope, choosing their sample size and then dividing their sample into the four strata. The following section briefly mentioned some of the auditors' various considerations when categorizing their findings. Chapter III then presented the categorization of data that ultimately led to the DODIG's most important finding - the projected monetary benefits of unnecessary purchase requests (\$68.2 million). Finally, the chapter concluded with NAVSUP's use of the audit results to report the budgetary impact for the following fiscal year.

Chapter IV presented an analysis of the data collected to make the audit conclusions. The first part of the chapter discussed the way the DODIG selected the "adjusted" universe, sample size and strata and their stated confidence level. The next section is the author's analysis of the sample size decisions and the author's investigation of alternate methods of selecting a sample size and stratifying the chosen sample. The chapter also presented and computed an aggregate estimated confidence interval based on the stratifications made during the audit and compared it to the stated accuracy of the DODIG report. The next section reviewed differing dollar figures used to make projections. In particular, this section explained how numbers changed as the audit progressed and how apparently no thorough final review was conducted to ensure that conclusions made early in the audit were based on the same figures as those used to make conclusions at the end of the audit.

B. CONCLUSIONS

During the DODIG audit , audited commands naturally put their emphasis on the individual findings. Each finding is closely scrutinized, verified, and argued over before

the auditors are allowed to report their result. What is clearly more difficult to argue about are the projections made after the audit findings are gathered. Even if the projections are explained, the precision of the findings is often beyond simple interpretations. Therefore, the auditors' report results tend to be the final word. The precision of the audit projections are critical because the audited commands can then more accurately interpret the audit findings.

During the process of reviewing the audit, results were found to be based on projections from the chosen sample. However, one of the unusual conclusions from the sample was that the universe size should be adjusted based on 21 sampled items valued at \$41.1 million that were determined to be outside the scope of the audit. The DODIG auditors projected that 374 items totaling \$171 million should be removed from the original universe. The result was an unorthodox "adjusted universe" of 1056 items with an estimated value totaling \$520.2 million. However, all subsequent calculations and estimates were based on the original population size of 1430. Because an accurate universe is essential for all sampling, the original universe should have been completely reviewed to remove items that were not to be included in the audit and the resulting universe size used in all the subsequent analyses.

Another finding of the audit review was that there was no documented procedure to explain why a sample size of 107 was chosen. Conversations with several of the DODIG statisticians revealed that no one could recall how a sample size of 107 was chosen. Despite the time between the audit and this thesis, 107 should have been

verifiable. This would have been possible if the procedure were documented, which it clearly was not. This issue is further discussed in the recommendations.

A significant finding of this thesis be found in Appendices B and M. These appendices show that 63.7% of the potential monetary benefits of unnecessary purchase requests were derived from the sample of the fourth stratum. That sample included only 3.4% ($44 + 1301$) of the stratum universe of purchase requests and only 3.0% ($10,736,025 + 358,632,509$) of the stratum universe's value. However, this same stratum included 91% of all purchase requests and nearly 52% of the purchase request value for the entire universe. In light of the very small sample used in the fourth stratum, the DODIG claim that their findings are based on a 95% confidence level with a stated 3% level of precision is definitely open to question.

A related conclusion of this thesis is that there was an arbitrary decision to divide the strata to ensure 100% inclusion of the highest dollar value purchase requests, 80% of the second highest strata, 30% of the next higher strata, and then pulling the remaining samples from the lowest dollar value strata. This focus on the dollar value of purchases created unrealistic assumptions about the number of line items considered unnecessary. Specifically, the DODIG's projected number of unnecessary purchase requests was 399 (see Appendix M). This conclusion assumes that nearly 28% ($399 + 1430$) of all ICP purchase requests were at least partially unnecessary. While this figure can be stated with 95% confidence, the wide range (e.g., lack of precision) of this interval must also be included when reporting such numbers. Instead, no precision was given for the number of unnecessary purchase requests.

A major finding of this thesis resulted from a test of the stated audit confidence level and precision of the dollar value estimates. While the audit results were said to have a precision of $\pm 3\%$ for the dollar value, the actual precision determined was $\pm 4.7\%$ or about 57% and \$2 million larger than stated. This is important because all the auditors' work was supposed to have led to a 95% confidence level and $\pm 3\%$ precision. The statistical accuracy of the DODIG findings is critical to anyone reviewing the report. Without this accuracy, the findings are continually open to questions.

C. RECOMMENDATIONS

This thesis has highlighted several potential opportunities for improving the outcome of DODIG audits and the confidence in the findings rendered.

1. DODIG Should Develop Standard Statistical Procedures and Publish Them

During conversations with several of the DODIG representatives, it became clear that audits such as those conducted on the Navy ICPs will be occurring with increased frequency. Naturally, the audited commands are sensitive to findings that put them in a negative light. This creates an incentive for the audited commands to criticize the way the auditors make their findings and conclusions. In order to avoid other reviews like this thesis, and also to simplify the process for the auditors, standardized procedures of statistical steps should be developed and eventually published. This method of full disclosure on the part of the auditors would benefit everyone in the audit process.

2. A Succinct Summary of the Audit Findings Should Be Provided to

Audited Commands

A thorough review of all the DODIG working papers and the final audit report clearly showed that, during the course of the audit, conclusions were reached and then changed because some of the early assumptions were changed. The result was that conclusions reached during various phases of the audit were based on differing assumptions. For example, the projections made for the SPCC portion of the premature purchase requests (Appendix K) were inexplicably made using an earlier result of an "excessive" value projection. However, throughout the rest of the audit, projections for all other groups were made using the original universe value. This small inconsistency created a difference in the two totals for "excessive" given in Columns (1) and (2) of Table VII. By requesting a summary of the conclusions, it would be easy for the audited commands to verify that all audit findings were based on the same set of facts.

3. Statistical Ranges Should Be Provided to Audited Commands With

Supporting Data

When commands are audited, they receive the audit results with a statement that the auditors have 95% confidence in their findings within a certain range or precision. This is normally a given that is not open to question. However, this critical figure should be fully explained. When only 3% of one stratum is reviewed and the audit findings from this stratum are used to extrapolate 63.7% of the findings, then it is essential to the audited commands that they have a complete understanding of why that was done. As mentioned earlier, the range of the audit findings are as important as the findings

themselves. Therefore, when a statement is made about "x" confidence level and "y" precision, these should be provided with the dollar values to which they refer. Then figures can be verified through the use of procedures like those used in Chapter IV.D. When this process is complete, a clear calculation of the precision is then available for review.

4. Validate Sample Sizes After the Audit

This thesis provides a process that can be used to determine the precision of selected sample sizes. For commands that are being audited for attributes only (unlike the ICPs), Chapter IV.C provides a process to verify the accuracy of the auditor's precision. Additionally, commands being audited for both variables and attributes can also use the process to ensure the sample size is statistically valid. If the audited command concludes the auditor's stated precision is inaccurate, the auditors should be approached immediately to allow them an opportunity to defend their findings or issue an addendum to the findings stated in the audit report.

5. Additional Research is Needed

The primary purpose of this thesis was to review the DODIG's sampling plan which was used in the evaluation of the two ICPs' purchase requests. What was not considered, but appeared to be an area of serious debate during the audit, were the issues of what constitute premature, unnecessary, or insufficient purchases. A major factor in this determination was the definition of "stockage objective." In order to narrow the scope of this thesis, these terms were not debated. However, correspondence between NAVSUP and the DODIG clearly showed that the definitions were open to interpretation.

A more in-depth look into this area would benefit the ICPs and others dependent on stockage objectives to effectively manage warehouse inventories.

GLOSSARY

Approved Acquisition Objective (AAO): The quantity of an item authorized for peacetime and wartime requirements to equip and sustain U.S. and Allied Forces in accordance with current DOD policies and plans.

Contingency Retention Stock (CRS): That portion of the quantity of an item greater than the AAO and economic retention stock for which there is no predictable demand or quantifiable requirement, and that would normally be allocated as Potential Reutilization Stock, except for a determination that the quantity will be retained for specific contingencies.

Due In Long Supply: Assets expected to arrive into the wholesale supply system in excess of all known or expected requirements during some time period, usually thought of as in excess of the Retention Limit.

Economic Retention Stock (ERS): That portion of the quantity of an item greater than the AAO determined to be more economical to retain for future peacetime issues than to dispose and satisfy projected future requirements through new procurement and/or repair. To warrant economic retention, an item must have a reasonably predictable demand rate.

Excessive: Purchase requests that fall into either the "premature" or "unnecessary" categories described below.

Hardware Systems Command: A headquarters activity that is responsible for the procurement and technical support of weapons system requirements (e.g., Naval Sea Systems Command and Naval Air Systems Command).

Non-projectable: A purchase request that could not be used to predict overall error because the ICPs had taken independent action to reduce excessive purchases. The actions were taken because of ICP item manager or supervisory review or an HSC had directed a curtailment to the purchase.

Potential Reutilization Stock: Material identified by an item manager for possible disposal but with potential for reutilization; or material that has the potential for being sent by an item manager to the Defense Reutilization and Marketing Service for possible reutilization by another Component or by a Federal, state, or local government agency.

Premature: A purchase request with a quantity that exceeds the stockage objective by more than twelve months of forecasted requirements.

Reasonable: Purchase requests that do not fit into the "excessive" or "undeterminable" definitions described here.

Retention Limit: The maximum quantity of on-hand material that may be retained in stock. This quantity is determined by summing AAO, ERS, and CRS.

Safety Level: The value of the expected net inventory (on hand minus backorders) just before an order arrives. This is a "cushion" of stock which is kept on hand to cover variations in demand during lead time.

Undeterminable: A purchase request for which DODIG was unable to determine the reasonableness because the ICPs could not provide verifiable requirements data as of September 30, 1990 or requirements were dependent on a critical management decision by a hardware systems command (HSC).

Unnecessary: A purchase request with a quantity in excess of five years of forecasted requirements.

APPENDIX A: NOTATION AND EQUATIONS USED IN STRATIFIED SAMPLING

Notations used in Stratified Sampling:

x_i = value of the "i"th observation

N = total number of units in all strata (the population)

c = coefficient of variation

s = standard deviation of a sample

\bar{X} = the mean of the x_i observations

L = number of strata into which the population is divided for purposes of stratified sampling

p = population proportion that possesses a specified characteristic

\hat{p}_x = an unbiased estimator of the population proportion p

p_i = the population proportion in stratum "i" that possesses a specified characteristic

\hat{p}_i = an unbiased estimator of p_i

N_i = total number of units in stratum "i"

n = number of units sampled out of the population

n_i = number of units sampled in stratum "i"

\hat{V} = estimated variance

$q = 1 - p$

$q_i = 1 - p_i$

i = stratum index

\bar{y}_i = sample mean of the i th stratum

\bar{y}_x = estimate of population mean (\bar{Y})

E = standard error

z = the normal deviate related to the confidence level (e.g., 1.96 for 95% confidence)

\hat{x} = mean estimator

(1) coefficient of variation, a measure of relative variability: (Neter, 1993, p. 84)

$$c = 100 \left(\frac{s}{\bar{X}} \right)$$

(2) estimator of the population proportion p: (Mendenhall, 1990, p. 117)

$$\hat{p}_x = \frac{1}{N} \sum_{i=1}^L N_i \hat{p}_i$$

(3) estimated variance of \hat{p}_x : (Mendenhall, 1990, p. 117)

$$\hat{V}(\hat{p}_x) = \frac{1}{N^2} \sum_{i=1}^L N_i^2 \left(\frac{N_i - n_i}{N_i} \right) \left(\frac{\hat{p}_i \hat{q}_i}{n_i - 1} \right)$$

(4) bound on the error of estimation: (Mendenhall, 1990, p. 117)

$$2\sqrt{\hat{V}(\hat{p}_x)} = 2 \sqrt{\frac{1}{N^2} \sum_{i=1}^L N_i^2 \left(\frac{N_i - n_i}{N_i} \right) \left(\frac{\hat{p}_i \hat{q}_i}{n_i - 1} \right)}$$

(5) standard error of a percent for a sample drawn from a finite population: (Hill, 1962, p. A-1)

$$\pm E = z \sqrt{\frac{pq}{n} \left(\frac{N-n}{N-1} \right)}$$

(6) variance of a sample: (Neter, 1993, p. 82)

$$s^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$$

(7) estimator for the population total audit value: (Raj, 1972, p. 53)

$$\hat{x} = \sum_{i=1}^L N_i \bar{y}_i$$

(8) estimated mean of the population: (Raj, 1972, p. 53)

$$\bar{Y} = \frac{1}{N} \sum_{i=1}^L N_i \bar{y}_i$$

(9) variance of the estimator \hat{x} : (Raj, 1972, p. 53)

$$s^2(\hat{x}) = \sum_{i=1}^L N_i^2 \left(\frac{1}{n_i} \right) \left(1 - \frac{n_i}{N_i} \right) s_i^2$$

(10) estimated standard deviation of the mean: (Neter, 1993, p. 291)

$$s\{\bar{x}\} = \frac{s}{\sqrt{n}}$$

APPENDIX B: SUMMARY OF UNIVERSE AND SAMPLE VALUES

ASO						
Stratum	Universe	Universe Value	Sample	Sample Value		
1	8	\$78,092,994	8	\$78,092,994		
2	26	83,895,837	20	64,160,301		
3	66	99,705,919	17	24,738,320		
4	789	234,191,420	24	5,648,744		
	889	\$495,886,170	69	\$172,640,359		
SPCC						
Stratum	Universe	Universe Value	Sample	Sample Value		
1	2	\$16,801,000	2	\$16,801,000		
2	6	19,377,237	6	19,377,237		
3	21	34,865,847	10	15,493,707		
4	512	124,441,089	20	5,087,281		
	541	\$195,285,173	38	\$56,759,225		
TOTAL						
Stratum	Universe	Universe Value	Sample	Sample Value		
1	10	\$94,893,994	10	\$94,893,994		
2	32	103,273,074	26	83,537,538		
3	87	134,371,766	27	40,232,027		
4	1301	358,632,509	44	10,736,025		
	1430	\$681,171,343	107	\$229,399,584		

APPENDIX C: COMPLETE LIST OF SAMPLED ITEMS (INITIALLY)

	Sample Numbers *		Extended Value	Subtotals
1	NA1-01		\$13,666,416	
2	NA1-02		11,967,280	
3	NA1-03		11,104,740	
4	NA1-04		10,634,238	
5	NA1-05		9,006,800	
6	NA1-06		7,935,475	
7	NA1-07		7,381,000	
8	NA1-08		6,397,046	
				\$78,092,995
9	NA2-01		2,902,650	
10	NA2-02		3,480,000	
11	NA2-03		3,112,110	
12	NA2-04		3,893,507	
13	NA2-05		3,035,373	
14	NA2-06		2,774,636	
15	NA2-07		2,602,380	
16	NA2-08		2,629,424	
17	NA2-09		4,916,280	
18	NA2-10		2,701,218	
19	NA2-11		3,141,600	
20	NA2-12		3,655,000	
21	NA2-13		2,569,869	
22	NA2-14		3,170,952	
23	NA2-15		3,272,160	
24	NA2-16		2,549,261	
25	NA2-17		2,827,366	
26	NA2-18		3,228,039	
27	NA2-19		3,007,680	
28	NA2-20		4,690,796	
				64,160,301
29	NA3-01		1,486,694	
30	NA3-02		1,241,267	
31	NA3-03		1,314,587	
32	NA3-04		1,269,709	
33	NA3-05		1,821,430	
34	NA3-06		1,673,360	
35	NA3-07		1,122,028	
36	NA3-08		2,340,030	
37	NA3-09		1,346,856	
38	NA3-10		1,141,950	
39	NA3-11		1,067,950	
40	NA3-12		1,146,958	
41	NA3-13		1,334,520	
42	NA3-14		1,159,297	
43	NA3-15		1,994,278	
44	NA3-16		1,254,300	
45	NA3-17		2,023,104	
				24,738,318

APPENDIX C: COMPLETE LIST OF SAMPLED ITEMS (INITIALLY)

46	NA4-01			411,350	
47	NA4-02			126,943	
48	NA4-03			587,070	
49	NA4-04			298,580	
50	NA4-05			163,895	
51	NA4-06			200,429	
52	NA4-07			180,000	
53	NA4-08			225,146	
54	NA4-09			117,539	
55	NA4-10			128,370	
56	NA4-11			227,125	
57	NA4-12			294,000	
58	NA4-13			434,968	
59	NA4-14			113,920	
60	NA4-15			313,200	
61	NA4-16			121,726	
62	NA4-17			113,971	
63	NA4-18			185,786	
64	NA4-19			340,000	
65	NA4-20			438,070	
66	NA4-21			106,587	
67	NA4-22			130,314	
68	NA4-23			186,480	
69	NA4-24			203,274	
					5,648,743
70	NS1-01			10,186,600	
71	NS1-02			6,614,400	
					16,801,000
72	NS2-01			3,840,000	
73	NS2-02			3,592,000	
74	NS2-03			3,150,000	
75	NS2-04			3,084,156	
76	NS2-05			3,045,690	
77	NS2-06			2,665,391	
					19,377,237
78	NS3-01			1,750,000	
79	NS3-02			1,602,685	
80	NS3-03			1,108,500	
81	NS3-04			1,150,000	
82	NS3-05			1,152,000	
83	NS3-06			1,102,000	
84	NS3-07			2,232,510	
85	NS3-08			2,100,000	
86	NS3-09			2,256,012	
87	NS3-10			1,040,000	
					15,493,707
88	NS4-01			225,215	
89	NS4-02			350,980	
90	NS4-03			145,000	

APPENDIX C: COMPLETE LIST OF SAMPLED ITEMS (INITIALLY)

91	NS4-04			214,773	
92	NS4-05			155,682	
93	NS4-06			109,152	
94	NS4-07			482,030	
95	NS4-08			610,977	
96	NS4-09			266,489	
97	NS4-10			144,750	
98	NS4-11			268,935	
99	NS4-12			170,880	
100	NS4-13			105,934	
101	NS4-14			125,844	
102	NS4-15			291,816	
103	NS4-16			731,766	
104	NS4-17			278,694	
105	NS4-18			129,422	
106	NS4-19			153,920	
107	NS4-20			125,022	
					5,087,281
					\$229,399,582
	* A = ASO				
	S = SPCC				
	1st Number = Stratum				
	2nd and 3rd Numbers = Number Within Stratum				

APPENDIX D: ADJUSTED PURCHASE REQUESTS

ASO							
Stratum		Adjustment to Sample Hits		Adjustment to Sample Hit Value		Projected Hits	Projected Value
1		1		\$9,006,800		1	\$9,006,800
2		5		17,394,859		6	22,737,337
3		2		2,601,156		8	10,483,782
4		7		1,717,478		230	72,117,091
		15		\$30,720,293		245	\$114,344,990
SPCC							
Stratum		Adjustment to Sample Hits		Adjustment to Sample Hit Value		Projected Hits	Projected Value
1		1		\$10,186,600		1	\$10,186,600
2		0*		(1,796,000)*		0	(1,796,000)
3		0		0		0	0
4		5		1,971,983		128	48,237,111
		6		\$12,158,583		129	\$56,627,711
TOTAL							
Stratum		Adjustment to Sample Hits		Adjustment to Sample Hit Value		Projected Hits	Projected Value
1		2		\$19,193,400		2	\$19,193,400
2		5		15,598,859		6	20,941,337
3		2		2,601,156		8	10,483,782
4		12		3,689,461		358	120,354,202
		21		\$41,082,876		374	\$170,972,701
*During the initial sample, item #NS2-02 was incorrectly recorded as two vice three. This adjustment increased the adjusted sample size but does not change the number of items sampled.							

APPENDIX E: COMPLETE LIST OF SAMPLED ITEMS (REVISED)

	Sample Numbers	National Stock Numbers	Quantity	Extended Value	Subtotals
1	NA1-01	1615-01-201-9639	94	\$13,666,416	
2	NA1-02	1650-01-277-8238	74	11,967,280	
3	NA1-03	2840-01-251-7227	51	11,104,740	
4	NA1-04	5960-LL-NEE-A190	253	10,634,238	
5	NA1-06	2840-LL-WY5-5024	16	7,935,474	
6	NA1-07	6605-01-054-3776	61	7,381,000	
7	NA1-08	1615-01-201-9601	44	6,397,046	
					\$69,086,194
8	NA2-01	1615-01-158-9678	111	2,902,650	
9	NA2-02	1650-01-161-4420	58	3,480,000	
10	NA2-04	2840-01-142-8818	298	3,893,507	
11	NA2-05	1560-01-155-7014	69	3,035,373	
12	NA2-06	5998-01-306-1972	17	2,774,636	
13	NA2-07	4920-01-124-9246	22	2,602,380	
14	NA2-08	4920-01-220-4520	119	2,629,424	
15	NA2-10	5865-01-196-9869	26	2,701,218	
16	NA2-14	1430-01-325-2512	80	3,170,952	
17	NA2-15	4920-01-279-8220	102	3,272,160	
18	NA2-16	5855-01-052-6849	11	2,549,261	
19	NA2-17	1620-00-761-4903	21	2,827,366	
20	NA2-18	4920-01-156-1393	27	3,228,039	
21	NA2-19	1270-01-256-8264	20	3,007,680	
22	NA2-20	4920-01-124-9245	31	4,690,796	
					46,765,442
23	NA3-01	1620-01-177-1891	61	1,486,694	
24	NA3-02	5895-01-303-7755	37	1,241,267	
25	NA3-03	1560-01-300-7768	18	1,314,587	
26	NA3-04	1740-01-062-1657	55	1,269,709	
27	NA3-05	5841-01-004-7531	65	1,821,430	
28	NA3-06	2835-01-256-8378	13	1,673,360	
29	NA3-07	2925-01-277-3508	72	1,122,028	
30	NA3-08	1630-01-106-4900	462	2,340,030	
31	NA3-10	1730-01-126-6239	30	1,141,950	
32	NA3-11	6720-01-181-5872	25	1,067,950	
33	NA3-12	7021-01-283-3749	6	1,146,958	
34	NA3-13	6605-01-245-8209	60	1,334,520	
35	NA3-14	6605-01-027-4172	25	1,159,297	
36	NA3-15	5963-01-154-2794	65	1,994,278	
37	NA3-17	1280-01-095-2982	24	2,023,104	
					22,137,162
38	NA4-01	6610-01-278-9291	19	411,350	
39	NA4-02	1560-01-284-5093	10	126,943	
40	NA4-03	6615-01-183-7413	99	587,070	
41	NA4-05	5999-01-271-1243	31	163,895	
42	NA4-07	4920-01-251-7174	3	180,000	
43	NA4-08	1610-00-887-0392	18	225,146	
44	NA4-10	2840-01-150-6734	33	128,370	
45	NA4-11	4920-01-054-9326	23	227,125	

APPENDIX E: COMPLETE LIST OF SAMPLED ITEMS (REVISED)

46	NA4-12	4810-01-271-8852	70	294,000	
47	NA4-15	6615-01-129-8410	9	313,200	
48	NA4-16	1280-01-186-1434	11	121,726	
49	NA4-18	1680-01-242-9698	11	185,786	
50	NA4-19	2840-01-281-3618	4	340,000	
51	NA4-21	7050-01-098-5523	1	106,587	
52	NA4-22	1680-01-159-9153	1	130,314	
53	NA4-23	6680-01-175-9116	21	186,480	
54	NA4-24	1620-01-158-5958	56	203,274	
					3,931,266
55	NS1-02	1720-01-271-1475	424	6,614,400	
					6,614,400
56	NS2-01	5895-01-281-2401	80	3,840,000	
57	NS2-02	2010-01-111-9593	3	5,388,000	
58	NS2-03	5960-01-302-4456	210	3,150,000	
59	NS2-04	6625-01-234-0485	171	3,084,156	
60	NS2-05	6625-01-146-1554	215	3,045,690	
61	NS2-06	6625-01-233-7104	154	2,665,391	
					21,173,237
62	NS3-01	6150-01-306-7242	5	1,750,000	
63	NS3-02	1420-01-108-5915	35	1,602,685	
64	NS3-03	5820-01-020-2762	5	1,108,500	
65	NS3-04	4310-01-187-5041	50	1,150,000	
66	NS3-05	5999-00-619-7838	24	1,152,000	
67	NS3-06	4320-01-220-1747	29	1,102,000	
68	NS3-07	6625-01-258-3140	70	2,232,510	
69	NS3-08	5845-01-307-6466	12	2,100,000	
70	NS3-09	6625-01-259-7355	29	2,256,012	
71	NS3-10	6695-01-299-8473	104	1,040,000	
					15,493,707
72	NS4-01	6130-01-155-2338	15	225,215	
73	NS4-02	5820-00-334-8407	70	350,980	
74	NS4-03	5999-01-255-1816	22	110,000	
75	NS4-04	2825-00-371-7899	13	214,773	
76	NS4-06	6605-01-030-0004	18	109,152	
77	NS4-07	2010-01-222-5283	4	482,030	
78	NS4-08	5985-01-119-3998	10	226,288	
79	NS4-10	4820-01-090-6529	50	144,750	
80	NS4-12	5915-00-527-9524	15	170,880	
81	NS4-13	4320-01-062-1473	34	105,934	
82	NS4-14	2010-01-144-2462	1	125,844	
83	NS4-15	6625-01-268-6800	27	291,816	
84	NS4-17	2865-01-164-1509	117	278,694	
85	NS4-19	2990-01-134-6899	88	153,921	
86	NS4-20	5998-01-183-7818	4	125,022	
	* A = ASO				3,115,299
	S = SPCC				\$188,316,707
	1st Number = Stratum				
	2nd and 3rd Numbers = Number Within Stratum				

APPENDIX F: SUMMARY OF ADJUSTED UNIVERSE AND SAMPLE VALUES

ASO							
Stratum		Universe		Universe Value		Sample	Sample Value
1		7				7	\$69,086,194
2		20				15	46,765,442
3		58				15	22,137,163
4		559				17	3,931,266
		644		0		54	\$141,920,065
SPCC							
Stratum		Universe		Universe Value		Sample	Sample Value
1		1				1	\$6,614,400
2		6				6	21,173,237
3		21				10	15,493,707
4		384				15	3,115,299
		412		0		32	\$46,396,643
TOTAL							
Stratum		Universe		Universe Value		Sample	Sample Value
1		8		\$75,700,594		8	\$75,700,594
2		26		82,331,737		21	67,938,679
3		79		123,888,003		25	37,630,870
4		943		238,278,307		32	7,046,565
		1056		\$520,198,641		86	\$188,316,708

APPENDIX G: UNDETERMINABLE PURCHASE REQUESTS

ASO							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		2		\$18,015,238		2	\$18,015,238
2		4		12,573,801		5	16,435,589
3		3		4,646,285		12	18,726,500
4		2		389,754		66	16,365,837
		11		\$35,625,078		85	\$69,543,164
SPCC							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		0		0		0	0
2		0		0		0	0
3		0		0		0	0
4		0		0		0	0
		0		\$0		0	\$0
TOTAL							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		2		\$18,015,238		2	\$18,015,238
2		4		12,573,801		5	16,435,589
3		3		4,646,285		12	18,726,500
4		2		389,754		66	16,365,837
		11		\$35,625,078		85	\$69,543,164

APPENDIX H: REASONABLE PURCHASE REQUESTS

ASO							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		3		\$31,007,495		3	\$31,007,495
2		6		25,869,400		8	33,814,662
3		5		10,827,385		19	43,638,953
4		5		2,376,480		164	99,788,676
		19		\$70,080,760		195	\$208,249,786
SPCC							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		1		\$614,400		1	\$6,614,400
2		5		17,717,238		5	17,717,238
3		6		11,560,810		13	25,866,326
4		7		1,578,600		179	38,614,479
		19		\$31,471,048		198	\$88,812,443
TOTAL							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		4		\$31,621,895		4	\$37,621,895
2		11		43,586,638		13	51,531,900
3		11		22,388,195		32	69,505,279
4		12		\$71,659,360		343	138,403,155
		38		\$169,256,088		392	\$297,062,229

APPENDIX I: EXCESSIVE PURCHASE REQUESTS

ASO							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		2		\$20,063,461		2	\$20,063,461
2		5		8,322,245		7	10,878,254
3		7		6,663,495		27	26,856,710
4		10		1,165,031		329	48,919,781
		24		\$36,214,232		365	\$106,718,206
SPCC							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		0		\$0		0	\$0
2		1		3,456,000		1	3,456,000
3		4		3,932,894		8	8,799,515
4		8		1,536,701		205	37,589,578
		13		\$8,925,595		214	\$49,845,093
TOTAL							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		2		\$20,063,461		2	\$20,063,461
2		6		11,778,245		8	14,334,254
3		11		10,596,389		35	35,656,225
4		18		2,701,732		534	86,509,359
		37		\$45,139,827		579	\$156,563,299

APPENDIX J: NON-PROJECTABLE PURCHASE REQUESTS

ASO								
Stratum		Hits		Hit Value		Projected Hits		Projected Value
1		2		\$20,063,461		2		\$20,063,461
2		4		4,551,027		5		5,948,782
3		3		2,158,877		12		8,701,188
4		3		269,893		99		11,332,857
		12		\$27,043,258		118		\$46,046,288
SPCC								
Stratum		Hits		Hit Value		Projected Hits		Projected Value
1		0		\$0		0		\$0
2		0		0		0		0
3		0		0		0		0
4		0		0		0		0
		0		\$0		0		\$0
TOTAL								
Stratum		Hits		Hit Value		Projected Hits		Projected Value
1		2		\$20,063,461		2		\$20,063,461
2		4		4,551,027		5		5,948,782
3		3		2,158,877		12		8,701,188
4		3		269,893		99		11,332,857
		12		\$27,043,258		118		\$46,046,288

APPENDIX K: PREMATURE PURCHASE REQUESTS

A&O							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		0		\$0		0	\$0
2		3		648,080		4	847,125
3		3		714,254		12	2,878,745
4		5		343,855		164	14,430,106
		11		\$1,705,989		180	\$18,155,976
SPCC							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		0		\$0		0	\$0
2		1		192,000		1	192,000
3		3		1,094,791		6	2,299,061
4		7		448,304		179	11,476,593
		11		\$1,735,095		186	\$13,967,654
TOTAL							
Stratum		Hits		Hit value		Projected Hits	Projected Value
1		0		\$0		0	\$0
2		4		840,080		5	1,039,125
3		6		1,809,045		18	5,177,806
4		12		791,959		343	25,906,699
		22		\$3,441,084		366	\$32,123,630

APPENDIX L: UNNECESSARY PURCHASE REQUESTS

ASO							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		0		\$0		0	\$0
2		3		3,123,138		4	4,082,347
3		3		3,790,365		12	15,276,777
4		6		551,483		197	23,156,818
		12		\$7,464,986		213	\$42,515,942
SPCC							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		0		\$0		0	\$0
2		1		3,264,000		1	3,264,000
3		3		2,838,103		6	6,350,013
4		7		1,088,396		179	26,623,501
		11		\$7,190,499		186	\$36,237,514
TOTAL							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		0		\$0		0	\$0
2		4		6,387,138		5	7,346,347
3		6		6,628,468		18	21,626,790
4		13		1,639,879		376	49,780,319
		23		\$14,655,485		399	\$78,753,456

APPENDIX M: UNNECESSARY PURCHASE REQUESTS WITH PROJECTED MONETARY BENEFITS

ASO							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		0		\$0		0	\$0
2		3		2,844,247		4	3,717,800
3		3		3,223,739		12	12,993,037
4		6		489,192		197	20,541,240
		12		\$6,557,178		213	\$37,252,077
SPCC							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		0		\$0		0	\$0
2		1		2,615,904		1	2,615,904
3		3		2,426,021		6	5,428,015
4		7		938,215		179	22,949,891
		11		\$5,980,140		186	\$30,993,810
TOTAL							
Stratum		Hits		Hit Value		Projected Hits	Projected Value
1		0		\$0		0	\$0
2		4		5,460,151		5	6,333,704
3		6		5,649,760		18	18,421,052
4		13		1,427,407		376	43,491,131
		23		\$12,537,318		399	\$68,245,887

**APPENDIX N: SAMPLE SIZES FOR SAMPLING ATTRIBUTES; EXPECTED RATE OF OCCURRENCE
UNKNOWN (50%), CONFIDENCE LEVEL 95%**

Items in the Population	Sample Size for Reliability of:			
	$\pm 2\%$	$\pm 3\%$	$\pm 4\%$	$\pm 5\%$
200				
300				
400				196
500				217
1,000			375	278
1,500			429	306
2,000		696	462	322
2,500	1225	748	484	333
3,000	1334	787	500	341
3,500	1424	818	512	346
4,000	1500	842	522	350
4,500	1566	863	530	354
5,000	1622	879	536	357
6,000	1715	906	546	361
7,000	1788	926	553	364
8,000	1847	942	558	367
9,000	1895	954	563	368
10,000	1936	964	566	370
15,000	2070	996	577	375
20,000	2144	1013	583	377
25,000	2191	1023	586	378
50,000	2291	1045	593	381
100,000	2345	1056	597	383
Note: This table should be used only when the auditor is unable or unwilling to fix a maximum occurrence rate to be expected. This conservative approach will result in a much larger sample size than will be found in tables where an expected maximum rate is estimated.				
Source: Sampling in Auditing, Hill, 1962. p. A-1				

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